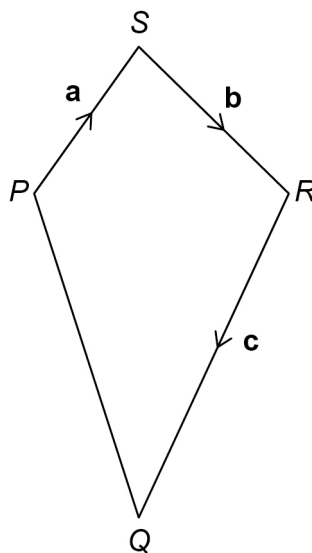


1 Here is quadrilateral $PQRS$.

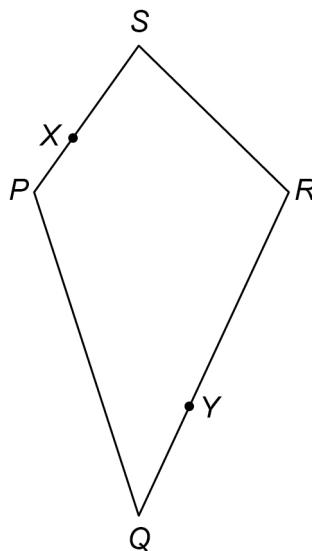
$$\overrightarrow{PS} = \mathbf{a} \quad \overrightarrow{SR} = \mathbf{b} \quad \overrightarrow{RQ} = \mathbf{c}$$



Not drawn
accurately

X is a point on PS where $PX : XS = 1 : 2$

Y is a point on RQ where $RY : YQ = 2 : 1$



Not drawn
accurately

Is XY parallel to PQ?

Show working to support your answer.

[3 marks]

$$\vec{PQ} = \vec{PS} + \vec{SR} + \vec{RQ}$$

$$= \underline{a} + \underline{b} + \underline{c} \quad (1)$$

$$\vec{XY} = \vec{XS} + \vec{SR} + \vec{RY}$$

$$= \frac{2}{3}\underline{a} + \underline{b} + \frac{2}{3}\underline{c} \quad (1)$$

No. as XY is not a multiple of PQ.

(1)

2

In the diagram

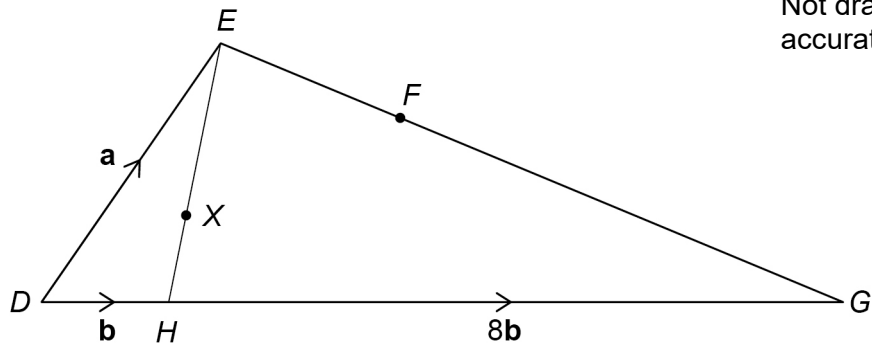
$$\overrightarrow{DE} = \mathbf{a}$$

$$\overrightarrow{DH} = \mathbf{b}$$

$$\overrightarrow{HG} = 8\mathbf{b}$$

$$EX : XH = 3 : 1$$

$$EF : FG = 1 : 3$$

Not drawn
accurately

2 (a)

Show that $\overrightarrow{DX} = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}$

[2 marks]

$$\overrightarrow{EH} = \overrightarrow{ED} + \overrightarrow{DH}$$

$$= -\mathbf{a} + \mathbf{b} \quad (1)$$

$$\overrightarrow{EX} = \frac{3}{4}(\overrightarrow{EH})$$

$$= \frac{3}{4}(-\mathbf{a} + \mathbf{b})$$

$$\overrightarrow{DX} = \overrightarrow{DE} + \overrightarrow{EX}$$

$$= \mathbf{a} + \left(-\frac{3}{4}\mathbf{a} + \frac{3}{4}\mathbf{b}\right) = \frac{1}{4}\mathbf{a} + \frac{3}{4}\mathbf{b} \quad (\text{shown})$$

2 (b)

Is DXF a straight line?

Show working to support your answer.

[4 marks]

$$\vec{EG} = \vec{EH} + \vec{HG}$$

$$= -\underline{a} + \underline{b} + 8\underline{b}$$

$$= -\underline{a} + 9\underline{b}$$

$$\vec{EF} = \frac{1}{4} \vec{EG} \quad (1)$$

$$= -\frac{1}{4}\underline{a} + \frac{9}{4}\underline{b} \quad (1)$$

$$\vec{DF} = \vec{DE} + \vec{EF}$$

$$= \underline{a} + \left(-\frac{1}{4}\underline{a} + \frac{9}{4}\underline{b}\right)$$

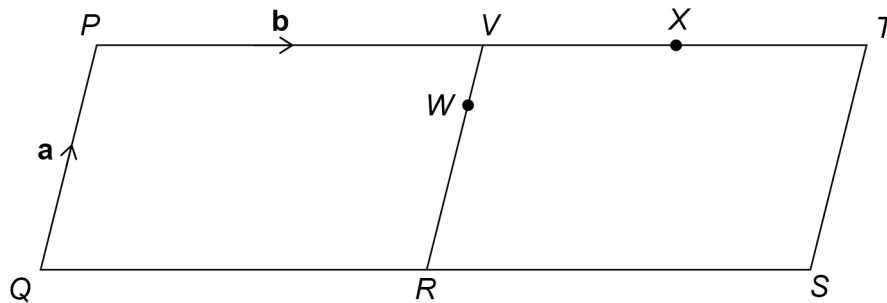
$$= \frac{3}{4}\underline{a} + \frac{9}{4}\underline{b} \quad (1)$$

$$\vec{DF} = 3\left(\frac{1}{4}\underline{a} + \frac{3}{4}\underline{b}\right)$$

$$\vec{DF} = 3(\vec{DX}) \quad (1)$$

Yes, DXF is a straight line.

3

Two congruent parallelograms, $PQRV$ and $VRST$, are joined.Not drawn
accurately

$$\overrightarrow{QP} = \mathbf{a} \quad \overrightarrow{PV} = \mathbf{b}$$

X is the midpoint of VT.

$$VW : WR = 1 : 2$$

Prove that Q, W and X lie on a straight line.

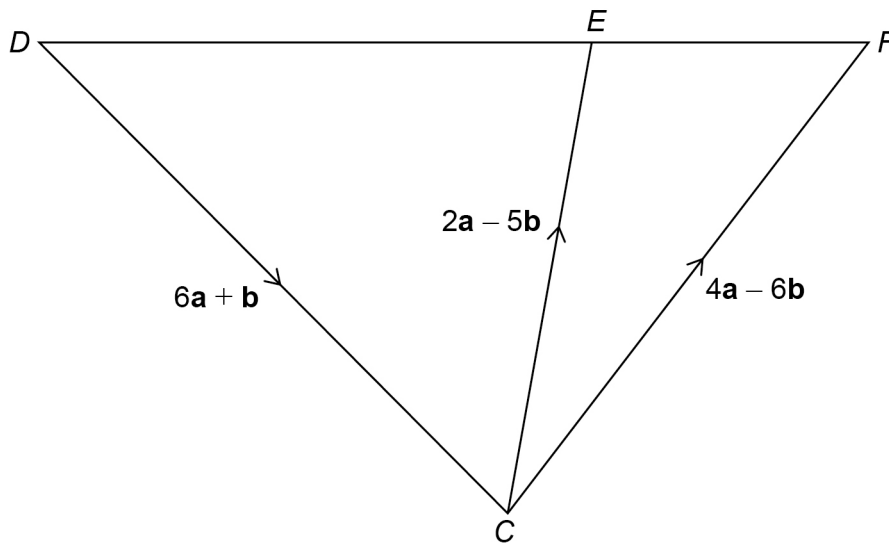
[3 marks]

$$\begin{aligned} \overrightarrow{QW} &= \overrightarrow{QP} + \overrightarrow{PV} + \overrightarrow{VW} \\ &= \underline{\mathbf{a}} + \underline{\mathbf{b}} + \frac{1}{3}(\overrightarrow{VR}) \\ &= \underline{\mathbf{a}} + \underline{\mathbf{b}} - \frac{1}{3}\underline{\mathbf{a}} \\ &= \frac{2}{3}\underline{\mathbf{a}} + \underline{\mathbf{b}} \quad (1) \end{aligned}$$

$$\begin{aligned} \overrightarrow{QX} &= \overrightarrow{QP} + \overrightarrow{PV} + \overrightarrow{VX} \\ &= \underline{\mathbf{a}} + \underline{\mathbf{b}} + \frac{1}{2}(\overrightarrow{VT}) \\ &= \underline{\mathbf{a}} + \underline{\mathbf{b}} + \frac{1}{2}\underline{\mathbf{b}} \\ &= \underline{\mathbf{a}} + \frac{3}{2}\underline{\mathbf{b}} \quad (1) \end{aligned}$$

$$\begin{aligned} \overrightarrow{QW} &= \frac{3}{2} \left(\frac{2}{3}\underline{\mathbf{a}} + \underline{\mathbf{b}} \right) = \underline{\mathbf{a}} + \frac{3}{2}\underline{\mathbf{b}} = \overrightarrow{QX} \\ \overrightarrow{QW} &= \frac{3}{2}\overrightarrow{QX} \quad (1) \end{aligned}$$

4

Not drawn
accuratelyProve that DEF is a straight line.

[4 marks]

$$\begin{aligned}
 \vec{DE} &= \vec{DC} + \vec{CE} \\
 &= 6\mathbf{a} + \mathbf{b} + 2\mathbf{a} - 5\mathbf{b} \\
 &= 8\mathbf{a} - 4\mathbf{b} \\
 &= 4(2\mathbf{a} - \mathbf{b}) \quad \text{✓ ①}
 \end{aligned}$$

$$\begin{aligned}
 \vec{DF} &= \vec{DC} + \vec{CF} \\
 &= 6\mathbf{a} + \mathbf{b} + 4\mathbf{a} - 6\mathbf{b} \\
 &= 10\mathbf{a} - 5\mathbf{b} \quad \text{✓ ①} \\
 &= 5(2\mathbf{a} - \mathbf{b}) \quad \text{✓ ①}
 \end{aligned}$$

$$\frac{\vec{DE}}{\vec{DF}} = \frac{4(2\mathbf{a} - \mathbf{b})}{5(2\mathbf{a} - \mathbf{b})} = \frac{4}{5} \quad \text{✓ ①}$$

Hence, \vec{DE} and \vec{DF} are parallel to each other.